between the duct walls. Experimental and numerical studies of these phenomena will be the subject of future research.

Acknowledgment

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Model for Compressible Turbulence in Hypersonic Wall Boundary and High-Speed Mixing Layers

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Introduction

THE most common approach to Navier-Stokes predictions of turbulent flows is based on either the classical Reynolds- or Favre-averaged Navier-Stokes equations or some combination.[1–3] Wilcox[4] suggests two research avenues for arriving at turbulence models suitable for hypersonic flows: 1) develop new models, or 2) generalize existing models. Wilcox,[4] as well as other researchers, have made great strides in the latter. As discussed by Wilcox, virtually all generalized models assume that Morkovin's hypothesis applies, which may become questionable for hypersonic boundary layers and high-speed free mixing layers. The present study addressed relaxing Morkovin's hypotheses by directly accounting for the density-velocity fluctuation correlation in the conservation equations of mass, momentum, and energy. The experimental results discussed later support this new ideology.

Mikulla and Horstman[5] present cross-wire measurements in a Mach 7 boundary layer. These data along with corollary data[6,7] were "re-reduced" here, using typical methods,[8] into the results given in Fig. 1a. The incompressible term of the Reynolds shear stress (Fig. 1a, open circles) only accounted for about 30% of the total turbulent shear stress (open triangles), and the compressible term (open squares) contributed the remaining 70%. Bowersox and Schetz[8] found that, for a Mach 4.0 free mixing layer, the incompressible shear term only accounted for about 25% (Fig. 2a, open circles) of the total shear stress (open triangles), whereas the compressible terms (open squares) contributed the remaining 75%.

The main goal of the current work was to numerically assess the effects of the compressible turbulence terms that were experimentally found to be important. The compressible apparent mass mixing length extension (CAMMLE) model,[8] which was based on measured experimental data (Figs. 1a and 2a, solid symbols), was found to produce accurate predictions of the measured compressible turbulence data for both the wall bounded and free mixing layer. Hence, that model was incorporated into a finite volume Navier-Stokes code.

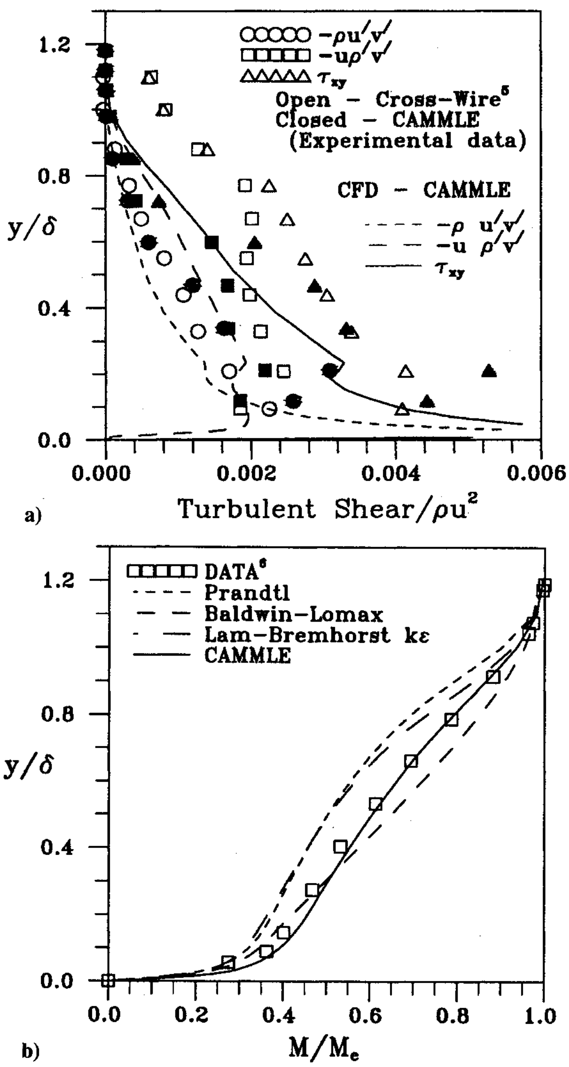

Fig. 1 Mach 6.85 (Refs. 5–7) experimental and numerical boundary-layer comparisons (x = 237 cm): a) Reynolds shear stress profiles (open symbols, cross-wire measurements; closed symbols, CAMMLE[8] model results evaluated with measured mean flow data; and lines, numerical CAMMLE results) and b) Mach number profile (symbols, measurements; and lines, numerical predictions).

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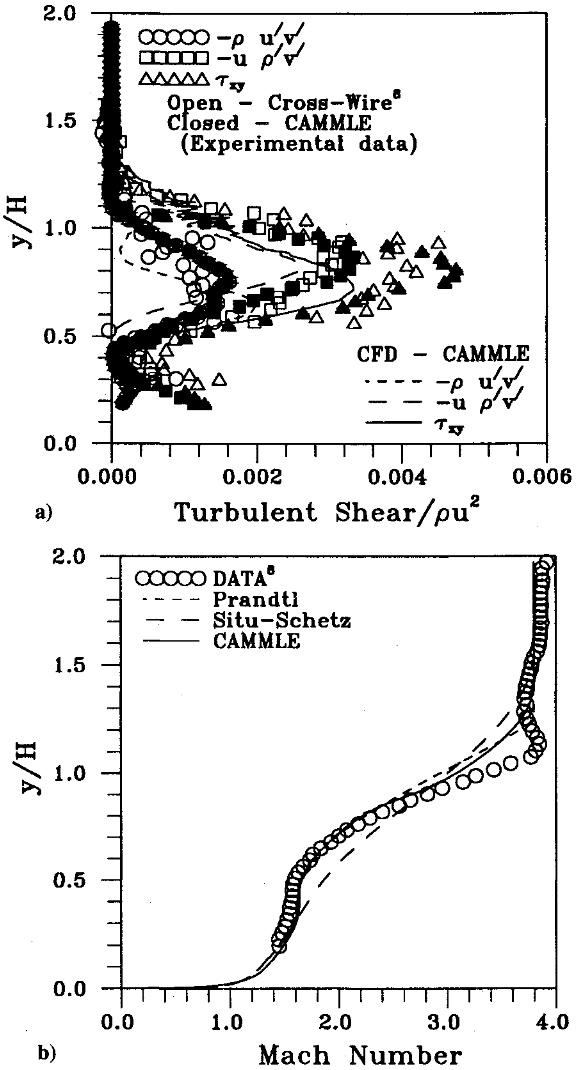

Fig. 2 Supersonic free mixing layer[8] experimental and numerical comparisons (x/H = 15): a) Reynolds shear stress profiles (nomenclature is the same as that of Fig. 1) and b) Mach number profile (nomenclature is the same as that of Fig. 1).

Turbulence Analysis

Applying Reynolds averaging to the conservative form of the Navier-Stokes equations, in cartesian coordinates, the turbulence terms for thin-layer-type flows can be written as

$$m_y^T = -\overline{\rho' v'}$$

$$\tau_{xy}^T = -\bar{\rho}\overline{u'v'} + \bar{u}m_y^T \qquad (1)$$

$$q_y^T = +\bar{\rho}\overline{h_0' v'} - \bar{h}_0 m_y^T$$

where m^T is the turbulent apparent mass (associated with continuity), τ^T is the Reynolds shear stress, and q^T is the compressible turbulent heat flux.

Generalizing the methods of Situ and Schetz,[2] the CAMMLE[8] model can be given by

$$m_y^T = \left|\frac{\partial\bar{\rho}}{\partial y}\right|\frac{\partial\bar{u}}{\partial y}\frac{L_m^2}{S}$$

$$\tau_{xy}^T = \mu_T\frac{\partial\bar{u}}{\partial y} + \bar{u}m_y^T \qquad (2)$$

$$q_y^T = -k_T\frac{\partial\bar{T}}{\partial y} - \bar{u}\mu_T\frac{\partial\bar{u}}{\partial y} - \bar{h}_0 m_y^T$$

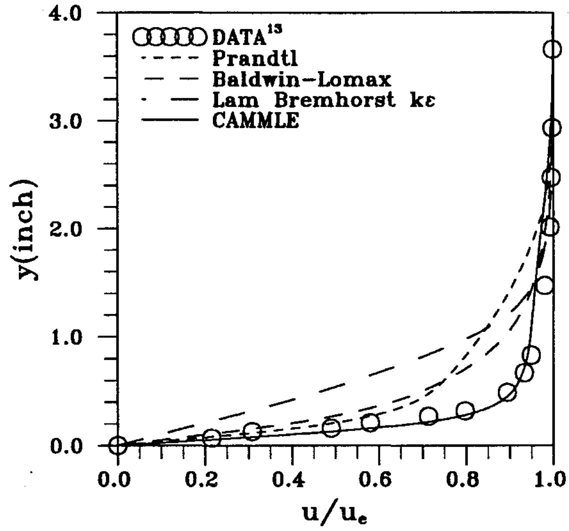

Fig. 3 Mach 20 (Ref. 13) experimental and numerical velocity profile comparisons (x = 139 in.).

where h_0' was expanded into $h' + \bar{u}u'$, μ_T is the original Prandtl mixing length eddy viscosity,[9] and $k_T = \mu_T Cp/Pr_T$. The CAMMLE model was derived following Prandtl and extending that idea to ρ'. Prandtl's mixing length theory can be obtained from the boundary-layer turbulent kinetic energy equation by considering the special case of negligible convection and diffusion. However, originally the method was based mainly on physical arguments and has provided useful results in a variety of flows.[10] Other researchers[3,4] have accounted for the density-velocity fluctuation correlation in the momentum equation. However, here the compressible terms were included in the conservation equations of mass, momentum, and energy. This ideology is supported by the experimental data of Bowersox and Schetz.[8]

Typical incompressible methods were used to determine the mixing length.[10] Situ and Schetz[2] set the new constant S equal to the turbulent Schmidt number. Li and Nagamatsu[11] defined a wall shear formulation similar to Eq. (2), where the constant was determined to be a function of Mach number. Bowersox and Schetz[8] experimentally determined that S = 1.0 agreed with the free shear layer turbulence data (Fig. 2a, solid symbols). The present study found that S = 2.5 produced accurate results for the hypersonic wall data presented in Fig. 1a (solid symbols). The different values of S at first appeared to be problematical; however, this may be a manifestation of Bradshaw's conclusion[9] concerning the compressibility in free mixing layers. Also, the two-equation extension method of Zeman, as reported in Wilcox,[4] incorporated different compressibility corrections for wall boundary and free shear layers.

Numerical Methods

The thin-layer parabolized Navier-Stokes equations with Roe's scheme[12] were solved. All solution normalized residuals were converged to 1×10^{-3} (i.e., three orders). Fine, medium, and course grids were used to insure grid convergence. Grid stretching was incorporated in shear regions, where for the medium grid the first cell height was chosen to correspond to $y^+ \approx 1$. For the free shear layer test case, the inflow boundary condition profile was generated by estimating the freestream and slot boundary-layer thicknesses from a shadowgraph[8] and using the typical velocity power law relation. For all of the boundary-layer calculations, the inflow was prescribed by a measured upstream turbulent profile. This methodology eliminated any uncertainty with transition location.

Results and Discussion

The first test case was a Mach 7.0 hypersonic air boundary layer.[6] The data were acquired on the constant diameter portion of the model; hence the pressure gradient effects were small and neglected. Since the diameter was much larger than the boundary-layer thickness, the flow was numerically treated as two-dimensional. The inflow profile was constructed from the $x = 115$ cm data. The CAMMLE shear stress predictions at $x = 237$ cm (Fig. 1a, lines) were slightly lower than the data; however, the model appeared to capture the trends of the turbulent flow physics. The CAMMLE model produced the best agreement with Mach number data (Fig. 2b). The Baldwin-Lomax, Prandtl, and k-ε models produced relatively poor results.

The second test case consisted of a hypersonic Mach 20 helium boundary layer.[13] Since the tunnel diameter was large compared with the wall boundary-layer thickness, the flow was treated as two dimensional in the numerical solutions. The inflow profile was constructed from the $x = 79$ in. data. The CAMMLE model velocity predictions at $x = 139$ in. (Fig. 3) are in excellent agreement with the data. The Baldwin-Lomax, Prandtl mixing length, and k-ε models were unsuccessful in predicting the "full" hypersonic velocity profile.

The last test case was a high-speed free mixing layer.[8] The numerical shear stress profiles at $x/H = 15$ (Fig. 2a) are reasonably predicted, again capturing the compressible turbulent flow trends. The CAMMLE model Mach number profile (Fig. 2b) is in good agreement with the data except near the outer edge of the layer. It is suspected that the effects of intermittency may have been important.

To assess the effects of the compressible terms, the Situ-Schetz[2] and Prandtl solutions were generated with the mixing lengths fixed to the values predicted by the CAMMLE model. The CAMMLE and Prandtl model results both agreed fairly well with the data. The Situ-Schetz formulation[2] significantly overpredicted the shear layer spreading. This result seemed problematical at first, since the Situ-Schetz model shear stress formulation is identical to that of the CAMMLE model. Hence, accounting for the compressible turbulence in all of the conservation equations with the CAMMLE model produced the improved prediction of the shear layer spreading. This is an important result since practically all modern techniques[4] require some sort of artificial "fix" to predict the correct spreading.

Conclusions

The effects of numerically including the compressible apparent mass terms in all of the conservation equations were assessed. A straightforward gradient transport analysis was applied to model the additional apparent mass term. This new formulation was incorporated into a modern Navier-Stokes computational fluid dynamics code. The new model as well as other popular models of varying complexity were numerically tested against experimental data. The new model produced significantly improved results. Hence, the numerical results reinforced the experimental conclusion that compressible turbulence terms are important for hypersonic wall boundary layers and high-density gradient flows. Finally, the numerical simplicity of the CAMMLE model may provide an engineering use for cases where higher-order models are not numerically applicable.

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Penetration and Mixing of Gas Jets in Supersonic Cross Flow

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Introduction

THE flowfield resulting from transverse injection of a gas jet from a wall into a supersonic crossflow is of interest in a number of practical applications. In all of these, there is a need for an analysis that predicts the gross features of the flow with reliable accuracy at a reasonable computational cost. This led us to undertake an update of the simplified analysis JETPEN[1] developed earlier. The requirements were to permit treatment of injection at angles other than 90 deg and to include turbulent mixing into the plume after the Mach disk.

Analysis

JETPEN used the "effective back pressure" concept[2] that relates the behavior of the jet as it exits into a lower pressure supersonic crossflow to the well-documented case of an underexpanded jet into a quiescent fluid by a model for the average pressure in the surroundings defined as the effective back pressure p_{eb}. In the earlier work, the simple models $p_{eb} \approx 0.8\, p'_a \approx 2/3\, p'_{ta}$ were used, but these were developed for 90-deg injection. Here, the approach has been extended to more general cases. We will shortly introduce a correlation for the angle of the jet at the Mach disk δ_1, and the injection angle is δ_j. The effective back pressure is now modeled as an average of the static pressure in the approach flow p_a and the Newtonian impact theory prediction for the pressure on bodies inclined at δ_1 and δ_j, $p_{\delta 1}$ and $p_{\delta j}$, as $p_{eb} = (p_{\delta 1} + p_{\delta j} + 2p_a)/4$. The correlation for δ_1 used here is

$$\delta_1 = \delta_j - \left(\frac{q_a}{q_j}\right)^{1/4} \frac{180}{\pi} \sin(\delta_j) \qquad (1)$$

The centerline trajectory of the jet to the Mach disk is taken as a parabola with δ_1 as the angle at the Mach disk. Also, we now choose to correlate the arc length along the trajectory to the Mach disk s rather than the vertical height of the Mach disk y_1 used before. The relation adopted is

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